Mortgage Option-Adjusted Term Structure Model (MOATS Model)

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Modeling Mortgage Rates[1]

For the valuation of mortgage-backed securities, investors have at their disposal sophisticated term structure models and prepayment models. The term structure model, calibrated to market interest rates and volatilities, gives a distribution of hypothetical paths of future benchmark interest rates. These benchmark rates are used to project future mortgage rates along the paths. These mortgage rates are then used by the prepayment model to project the prepayments and cash flows along the hypothetical paths. When the cash flows are in turn discounted by the corresponding short rates plus some spread, the resulting present values for the paths have a mean that represents the model price. The option-adjusted spread (OAS), defined as that spread for which the model price is the same as the market level, provides a uniform measure with which to compare securities having cash flows with diverse option characteristics.

Because the projected mortgage rate is the crucial factor affecting projected prepayment rates, its accuracy is of utmost importance in the pricing of mortgage-backed securities. The translation of benchmark rates to mortgage rates is, however, usually done in an ad hoc fashion. The common method is simply to select one benchmark interest rate and assume that the mortgage rate will always maintain today's spread over this selected benchmark, despite significant differences in projected yield curve levels, shapes and volatilities. The usual choice of this benchmark rate is the ten-year rate, specifically the ten-year Treasury rate. Although this surrogate mortgage rate has served well in the past, there is room for improvement.

The principal problem with the common method is that the current-coupon mortgage rate, as determined from the secondary market, is actually sensitive to the entire yield curve, not just the ten-year rate. For example, when the yield curve flattens between the short end and the ten-year point, TBA prices drop on account of the higher discounting rates. Accordingly, we expect the mortgage rate then to widen relative to the ten-year. It is important to incorporate such a dependency of projected mortgage rates on projected yield curves in our term structure model.

We have implemented a method, based on the arbitrage-free principle, in which mortgage rates are obtained at a constant OAS to projected yield curves, regardless of their level, shape, or volatility. The rest of this note will elaborate on this new model.

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Simple Spread or OAS?

It is unsatisfactory to derive the mortgage rate as a constant spread to any single key rate. At the very least, this would be inconsistent with the assumption that the current-coupon mortgage has a constant OAS. Assuming a constant OAS, a yield-curve reshaping or volatility change--even as the designated key rate remains fixed--would imply a change in prices of the TBAs, hence a change in the current-coupon spread.

The assumption of a simple constant mortgage spread over a single benchmark, be it Treasury or swap, ignores all these other likely future signals to the home-owner from the secondary market via the mortgage rate. In particular, it fails to estimate properly the key-rate partial durations important in the hedging and risk management of a mortgage position.

It is true that inconsistency with the constant OAS assumption does not make the constant spread assumption wrong; one could argue that it is the former that should be abandoned. Recent statistics indicate otherwise. For the period since January 1999, the standard deviation of the current-coupon OAS to the swap curve is 7.5bp compared to 15.4bp for the simple spread over the ten-year swap rate.

Mortgage Option-Adjusted Term Structure Model (MOATS)

Instead of calculating the current-coupon mortgage rate that drives prepayments as a spread to a single key rate, we have incorporated an arbitrage-free model that calculates these mortgage rates as part of the term structure model. This method uses a constant OAS to calculate mortgage prices and the current-coupon mortgage rates independently for each of the yield curves of various levels and slopes at each time step into the future. This is accomplished by a backward-induction method that accounts for the prepayment option of the mortgage holders should rates decline. At each time step, all later current-coupon rates are known, so that future prepayments can be determined. For any yield curve at that point in time, we can price any bond whose future cash flows are determined solely by the future path of the yield curve and are independent of the past. In particular, we can price new 30-year mortgages of any coupon. The coupon that has the price of par is, by definition, the current coupon. We use a simplified, path independent, version of the SSB prepayment model for the required path independence. The array of future mortgage current coupons is, however, calibrated to the initial market current coupon or its OAS using the full SSB path-dependent prepayment model. See the appendix for a brief description of the logic flow.

There is a marked effect on the volatility of the mortgage rate from using the MOATS approach. The mortgage rate inherits volatility also from rates shorter than the ten-year, since the current-coupon mortgage has duration closer to five years. These shorter rates, for example, the five-year rate, typically are more volatile, which will have a negative impact on mortgage values. The MOATS method also necessitates the inclusion of some swaptions of five-year tenor in our term-structure calibration; they had previously been ignored in the model where the ten-year rate is the all-important one for mortgages.

Treasury or Swap as Reference Curve?

The MOATS model assumes that the mortgage rate maintains a constant OAS, but which reference curve should be used? Any one could be used, but recent market history indicates that the swap curve serves best. A problem with the Treasury curve arose relatively recently when trading in Treasury securities became more technical as a result of the debt paydown. The mortgage market became more tightly linked to the swap market. Thus, the mortgage current-coupon spread over swap rates is now significantly more stable than the spread over Treasury rates. For example, for the spread of the 30-year conventional mortgage rate over the 10-year Treasury rate, the standard deviation is 28.1 bp for the period from January 1, 1996, to present. In contrast, for the spread over the 10-year swap rate, the standard deviation is 11.5 bp for the same period.

With the prospect of dwindling supply in Treasury bonds and notes, the distinction takes on some urgency. As forward swap spreads become more volatile, projection of mortgage cash flows based on forward 10-year Treasury rates becomes suspect. In particular, an inversion of the Treasury curve (e.g., September 27, 2000) likely understates the forward mortgage rates, and overestimates the in-the-moneyness of the prepayment options as a result. This unduly penalizes all mortgages, but especially the premium and cuspy coupons. Similarly, a steepening of the forward Treasury rates against the swap curve (e.g. the forward ten-year rates up to 10 years out on January 12, 2001) does the opposite. Accordingly, we prefer the swap curve as a basis to project mortgage rates.

There is however a side effect in using swap rates to project mortgage rates. In a model that is calibrated to swaption volatilities and that gives the Treasury rates roughly the same percentage volatilities as the corresponding swap rates, the absolute (basis-point) volatilities of the Treasury rates have been understated. Partially offsetting this are the overstated swaption volatilites for long expirations used as defaults in our model; they are currently set to a percentage of the 5-year volatility. When we replace the default values with actual market quotes for these swaptions, now more liquid than when we first developed our term structure model, lower long-term volatilities of the 10-year rate result.

In terms of mortgage rates, a swap-based model would lead to more volatile cash flows and richer valuation of all mortgage-backed securities that are short a prepayment option, especially discount pass-throughs with relatively long expected weighted-average lives and hence short a prepayment option with a high time value. Partially offsetting this is the lower long-term volatilities in the new model, especially for discounts.

Summing up, despite the Government or Agency credit quality, mortgages trade cheap to Treasuries in sympathy to the swap spread, and trade around the latter at least until some crisis prompts a flight to quality. A corollary is that swap rates also provide a better basis for the projection of mortgage rates and the mortgage prepayments than Treasury rates.

We emphasize that after the mortgage rates, and cash flows, are projected in the model, there is nothing to prevent an investor with Treasury benchmarks to discount the cash flows with Treasury rates. We emphasize also that the MOATS method could equally well be used to project mortgage rates with Treasury rates.

Impact on OAS of the MOATS Model

The impact on OAS of the new method depends on the yield curve and volatility environment. As illustration, consider two dissimilar dates. On October 11, 2000, the forward 10-year swap spread to the Salomon Smith Barney Treasury Model curve generally widens, and the volatilities are moderate. In comparison, on January 12, 2001, the forward 10-year swap spread tightens out to 10-years before widening, and the volatilities are high, except the very long-dated swaptions. We examine the impact of the model change on both dates.

Consider first the trade date October 11, 2000. We divide the change into three steps and display the changes in each step in Figure 1. In Figure 1, at the left we show the original OAS (swap curve discount). Next to that, we show the first step. If we simply add the widening in the forward ten-year swap spread (from its initial level) to the mortgage rates in the current model, the OAS increases, especially for premium coupons, since the prepayment option becomes less in the money. Next to that we show the second step, in which we switch to a constant spread over the ten-year swap rate. This results in more volatile mortgage rates and lower OASs for all pass-throughs. Finally, at the right, we see the third and final step, in which the current-coupon OAS remains constant. We see that MOATS lowers the OAS by another 2 to 5 bp, as the mortgage rate picks up the volatility of the shorter rates.

The combined effect, shown at the far right, is that pass-throughs have lower OASs by about 4 bp in the new model as compared to the old.

Figure 2 shows a similar pattern for strip IOs in each of the three steps, much more exaggerated in each step as IOs are leveraged in the prepayment risk. We note that the relative-value ranking of the securities is unchanged.

Figure 1. Impact of Different Mortgage Current-Coupon Models on Pass-Through OAS (OAS to swap, October 11, 2000)

Treasury Rates	2-Yr	5-Yr	10-Yr	30-Yr	Swap Spread	2-Yr	5-Yr	10-Yr	30-Yr
	5.94%	5.81	5.78	5.83		78.3 bp	99.4	118.9	120.0

Swaption Volatilities	3Mx10	1x10	3x10	5x10	7x10	10x10	20x10
	12.00%	13.9	14.2	13.2	12.3 (12.6 in old model)	10.35 (11.9 in old model)	6.5 (8.7 in old model)

	Original	Step One		Step Two		Step Three		
	Const CC Sprd to Treas 10 (Old Model)	CC Sprd Widens like Frwd 10-yr Swap-Sprd		Const CC Sprd to Swap 10		MOATS (New Model)		Total Change
	OAS	OAS	Δ(OAS)	OAS	Δ(OAS)	OAS	Δ(OAS)	Δ(OAS)
Conventional								
6.0 %	-1 bp	1 bp	2 bp	-4 bp	-4 bp	-5 bp	-2 bp	-4 bp
6.5	-6	-2	3	-7	-5	-10	-3	-4
7.0	-12	-8	4	-12	-4	-15	-4	-3
7.5	-7	-3	5	-7	-4	-11	-4	-4
8.0	-3	2	5	-2	-4	-6	-5	-4
8.5	1	5	5	2	-3	-2	-5	-3
Ginnie Mae								
6.0 %	-18 bp	-16 bp	2 bp	-21 bp	-5 bp	-24 bp	-3 bp	-6 bp
6.5	-15	-12	3	-18	-6	-22	-4	-6
7.0	-15	-10	4	-1 6	-6	-21	-5	-6
7.5	-14	-10	5	-15	-5	-20	-5	-5
8.0	-3	1	5	-3	-4	-8	-5	-5
8.5	-3	2	5	-1	-3	-6	-5	-4

Source: Salomon Smith Barney.

Figure 2. Impact of Different Mortgage Current-Coupon Models on Strip IO OAS (OAS to swap, October 11, 2000)

	Original	Step One		Step Two		Step Three		
	Const CC Sprd to Treas10 (Old Model)	CC Sprd Widens like Forwd 10-yr Swap-Sprd		Const CC Sprd to Swap 10		MOATS (New Model)		Total Change
Coupon	OAS	OAS	Δ(OAS)	OAS	Δ(OAS)	OAS	Δ(OAS)	Δ(OAS)
6.0 %	15 bp	87 bp	72 bp	-2 bp	-107 bp	-96 bp	-75 bp	-111 bp
6.5	47	131	84	9	-121	-78	-88	-125
7.0	25	124	99	4	-120	-89	-94	-115
7.5	85	182	97	77	-105	-15	-92	-100
8.0	230	327	96	256	-70	168	-89	-63

Source: Salomon Smith Barney.

The model impact on OAS is very different in the environment of January 12, 2001.

The changes in OAS are again displayed, in Figure 3 and Figure 4, in three steps. This time, the tightening ten-year swap spread makes all prepayment options more in-the-money, hurting all pass-throughs and especially IOs in Step One. At the same time, while the higher mortgage volatilities inherited from shorter-dated swaptions hurt some, the kinder and gentler volatilities for long-dated swaptions in the new model actually help those securities that are expected to live that long. Thus the change in Step Two makes the passthroughs and even some low-coupon IOs look cheaper. Changes due strictly to the MOATS method, displayed in Step Three, are comparable to those in Figure 1 and Figure 2.

Figure 3. Impact of Different Mortgage Current-Coupon Models on Pass-Through OAS (OAS to swap, January 12, 2001)

Treasury Rates	2-Yr	5-Yr	10-Yr	30-Yr		Swap Spread	2-Yr	5-Yr	10-Yr	30-Yr
	4.88%	4.95	5.25	5.62			66.8bp	90.9	89.2	73.1
Swaption Volatilities	3Mx10	1x10	3x10	5x10	7x10		10x10			20x10
	20.50%	17.4	16.3	14.9	13.4 (14.3 in old mode)l		11.2(13.4 in old model)			6.8(9.8 in old model)

Original Const CC Sprd to treas 10 (Old Model)	Step One: CC Sprd Widens like Frwd 10-yr Swap-Sprd		Step Two: Const CC Sprd to Swap 10		Step Three: MOATS (New Model)		Total Change
OAS	OAS	Δ (OAS)	OAS	Δ (OAS)	OAS	Δ (OAS)	Δ (OAS)
Conventional							
6.5 -6.0 bp	-7 bp	-1 bp	-2 bp	5 bp	-6 bp	-4 bp	0 bp
7.0 -3.0	-4	-1	2	6	-2	-4	1
7.5 3	1	-2	8	7	3	-5	0
8.0 10	7	-3	14	7	8	-5	-1
8.5 7	3	-4	10	7	4	-5	-2
Ginnie Mae							
6.0 -12 bp	-12 bp	0 bp	-10 bp	1 bp	-15 bp	-4 bp	-3 bp
6.5 -10	-11	-1	-6	4	-11	-5	-2
7.0 -5	-6	-2	0	7	-5	-5	0
7.5 0	-2	-2	6	8	0	-6	0
8.0 21	18	-3	25	7	20	-5	-1
8.5 14	10	-4	18	8	12	-5	-1

Source: Salomon Smith Barney.

Figure 4. Impact of Different Mortgage Current-Coupon Models on Strip IO OAS (OAS to swap, January 12, 2001)

Original Const CC Sprd to Treas10 (Old Model) Coupon	OAS	Step One: CC Sprd Widens like Forwd10-yr Swap-Sprd OAS	Δ (OAS)	Step Two: Const CC Sprd to Swap10 OAS	Δ (OAS)	Step Three: MOATS (New Model) OAS	(OAS)	Total Change Δ (OAS)
6.0	215	166	-49	137	-30	56	-81	-159
6.5	192	119	-73	125	6	40	-85	-152
7.0	189	85	-104	153	68	54	-99	-136
7.5	222	110	-112	200	90	97	-103	-125
8.0	446	329	-117	446	117	354	-92	-92

Source: Salomon Smith Barney.

Figure 5 shows the impact of the model change for July 31, 2001. Because the forward swap spreads on this date widen more than on January 12, 2001, prepayments for the new model are smaller than for the old model and many OAS values increase. In addition, the relatively higher long-term volatilities cause higher projected mortgage rates in the new model, again contributing to milder prepayments.

Figure 5. Impact of Different Mortgage Current-Coupon Models on OAS (OAS to swap, July 31, 2001)

Treasury Rates	2-Yr	5-Yr	10-Yr	30-Yr	Swap Spread	2-Yr	5-Yr	10-Yr	30-Yr
	3.85 %	4.57 %	5.09 %	5.52 %		44.5bp	72.8	73.1	65.4

Swaption Volatilities	3Mx10	1x10	3x10	5x10	7x10	10x10	20x10
	18.70 %	18.0	16.95	15.95	14.75	12.70	7.50
	same in old model except:				15.29	14.36	11.63

Original	Old Model	Step Two		Step Three		
Const CC	Const CC Sprd	Const CC Sprd		MOATS		Total
Sprd to treas 10 (Old Model)	to Treasury 10	to Swap 10		(New Model)		Change
		OAS	Δ (OAS)	OAS	Δ (OAS)	Δ (OAS)
Conventional						
6.0%	1 bp	3 bp	+2 bp	1 bp	-2 bp	0 bp
6.5	6	9	+3	7	-2	+1
7.0	7	12	+5	10	-2	+3
7.5	5	11	+6	11	0	+6
8.0	14	20	+6	21	+1	+7
8.5	3	8	+5	9	+1	+6
GNMA						
6.0%	-25 bp	-24 bp	+1 bp	-26 bp	-2 bp	-1 bp
6.5	-15	-12	+3	-15	-3	0
7.0	-12	-7	+5	-9	-2	+3
7.5	-5	1	+6	0	-1	+5
8.0	-2	5	+7	5	0	+7
8.5	11	17	+6	17	0	+6
IO Strips						
6.0%	76 bp	32 bp	-44 bp	37 bp	+5 bp	-39 bp
6.5	122	93	-29	113	+20	-9
7.0	346	349	+3	404	+55	+58
7.5	435	443	+8	505	+62	+70
8.0	795	813	+18	891	+78	+96

Source: Salomon Smith Barney.

Impact on Duration and Partial Duration

Figure 6 displays the effective and partial durations of mortgage pass-throughs and strips. The new model has little impact on the effective duration of pass-throughs. POs backed by discount coupons have somewhat longer effective durations in the new model; the opposite holds for those backed by premiums. This is because out-of-money call options, and out-of-money prepayment options in particular, have increasing dv01[2] as the volatility increases. The opposite is true for in-the-money options. As the mortgage rate volatility is higher in MOATS, so the POs, as options on mortgage rates will have higher or lower duration depending on whether the collateral coupon is out of or in the money.

More drastic are the changes in partial durations. The 10-year partial duration for Coupon 7.0% PO decreases from 13.9 to 9.3 while the 5-year and 30-year partial durations increase. This is because in the old model, an increase in the 10-year rate, even as the 5-year and 30-year rates remain unchanged, implies an increase in the mortgage rate of equal magnitude, slowing down prepayment. In MOATS, the mortgage rate is affected only partly by the 10-year rate; the fact that the 5- and 30-year rates remain low leads to a mortgage-rate increase that is more subdued than in the old model. The slowdown in prepayment is accordingly smaller in magnitude. Hence the smaller 10-year partial duration in MOATS. Similar reasoning explains the increase in 5- and 30-year partial durations for POs. The 2-year partial durations for POs with collateral coupon under 7.5% are lower (algebraically) in the new model. The reason is that an increase in the 2-year rate, with the other key rates fixed, results in a decrease in forward 5-year rates for up to five years. Since in MOATS the mortgage rate has a 5-year component, we have a decrease in forward mortgage rates. The faster forward prepayment benefits POs of longer expected average life, more than compensating for the higher discounting short rates.

Partial durations for Strips relative to swap spreads, displayed under the headings "Sw2Y" and "Sw5Y", are generally larger in magnitude than in the old model. That is simply because in the old model the swap spreads affected the model prices only through the discounting rates; they played no role in prepayment.

[2] Increase in call price when rates drop one basis point.

Figure 6. Effective Duration and Partial Durations of Pass-Throughs (October 11, 2000)

Coupon		Effective Duration	2 YR	5 Yr	10 Yr	30 Yr	Sw2Y	Sw5Y	Cap2Y	Cap5Y	Swn1x10	Swn5x10
Conventional												
6.0	**Old**	5.4	0.5	1.2	2.9	0.9	0.5	4.9	-0.0	0.0	0.0	0.2
	New	5.4	0.5	1.2	2.9	0.8	0.5	5.0	0.0	0.1	0.0	0.2
6.5		4.9	0.6	1.2	2.4	0.7	0.6	4.6	0.0	0.0	0.0	0.2
		4.9	0.6	1.2	2.4	0.7	0.6	4.3	0.0	0.1	0.0	0.2
7.0		4.4	0.7	1.2	1.9	0.6	0.6	4.2	0.0	0.0	0.1	0.2
		4.3	0.7	1.0	2.1	0.5	0.7	3.7	0.0	0.0	0.1	0.2
7.5		3.7	0.8	1.2	1.3	0.5	0.7	3.7	0.0	0.0	0.1	0.2
		3.7	0.8	0.9	1.6	0.4	0.8	3.0	0.0	0.1	0.1	0.1
8.0		2.9	0.8	1.1	0.7	0.4	0.7	3.2	0.0	0.1	0.1	0.1
		2.9	0.8	0.8	1.2	0.2	0.8	2.3	0.0	0.1	0.1	0.1
8.5		2.1	0.8	0.9	0.2	0.2	0.7	2.4	0.0	0.1	0.1	0.1
		2.1	0.7	0.6	0.7	0.1	0.8	1.4	0.0	0.1	0.1	0.1
Ginnie Mae												
6.0	**Old**	6.2	0.4	1.0	3.4	1.3	0.4	5.5	-0.0	0.0	0.0	0.2
	New	6.1	0.4	1.1	3.4	1.2	0.4	5.7	0.0	0.1	0.0	0.2
6.5		5.4	0.5	1.1	2.7	1.0	0.5	5.0	0.0	0.0	0.0	0.2
		5.4	0.6	1.1	2.8	0.9	0.5	4.9	0.0	0.1	0.0	0.2
7.0		4.6	0.7	1.1	2.0	0.8	0.6	4.5	0.0	0.0	0.0	0.2
		4.6	0.7	1.0	2.2	0.7	0.7	4.0	0.0	0.1	0.1	0.2
7.5		4.1	0.7	1.2	1.5	0.7	0.6	4.3	0.0	0.0	0.1	0.2
		4.0	0.8	0.9	1.9	0.5	0.8	3.4	0.0	0.1	0.1	0.2
8.0		3.4	0.8	1.2	1.0	0.5	0.7	3.8	0.0	0.1	0.1	0.2
		3.4	0.8	0.9	1.5	0.3	0.8	2.8	0.0	0.1	0.1	0.1
8.5		2.3	0.8	1.0	0.3	0.3	0.7	2.8	0.0	0.1	0.1	0.2
		2.3	0.7	0.6	0.8	0.1	0.8	1.7	0.0	0.1	0.1	0.1
PO (Conventional Collateral)												
6.0	**Old**	12.1	-0.6	0.1	9.5	3.0	0.5	5.2	0.0	-0.2	-0.0	-0.1
	New	13.0	-1.4	1.7	8.3	4.4	-1.0	13.3	0.0	-0.1	0.0	0.0
6.5		13.0	-0.7	0.3	10.7	2.8	0.6	4.6	0.0	-0.2	-0.1	-0.1
		13.4	-1.3	2.2	8.5	4.2	-1.1	13.6	0.0	-0.1	0.0	0.1
7.0		16.5	-0.6	0.3	13.9	2.8	0.7	4.0	0.0	-0.3	-0.1	0.0
		15.5	-0.7	2.9	9.3	4.1	-1.0	15.1	-0.1	-0.1	-0.1	0.5
7.5		16.6	-0.3	0.4	14.0	2.5	0.8	3.6	0.0	-0.3	-0.1	0.1
		15.5	-0.1	3.0	9.0	3.7	-0.6	14.8	0.0	-0.1	0.0	0.5
8.0		16.9	0.1	0.3	14.2	2.3	0.8	3.1	0.0	-0.3	0.1	0.1
		15.9	0.4	3.1	8.9	3.4	-0.4	14.6	0.0	-0.1	0.1	0.6
IO (Conventional Collateral												
6.0	**Old**	-7.7	2.5	3.2	-10.2	-3.2	0.5	4.3	-0.0	0.4	0.2	0.7
	New	-10.2	4.3	0.5	-8.3	-6.8	3.7	-12.2	0.0	0.4	0.1	0.4
6.5		-12.3	3.3	3.1	-15.0	-3.6	0.5	4.1	0.0	0.5	0.2	0.8
		-13.9	4.8	-0.8	-10.8	-7.3	4.3	-16.1	0.0	0.3	0.1	0.1
7.0		-22.7	3.6	3.0	-24.9	-4.2	0.6	4.0	0.0	0.8	0.4	0.5
		-21.6	4.1	-3.0	-14.6	-8.0	4.6	-22.8	0.1	0.5	0.3	-0.6
7.5		-27.2	3.3	2.7	-28.9	-4.0	0.6	3.7	0.0	0.8	0.4	0.4
		-25.4	3.1	-4.2	-16.5	-7.9	4.3	-25.9	0.1	0.4	0.3	-0.8
8.0		-34.3	2.5	2.6	-35.2	-3.9	0.6	3.3	0.0	0.8	0.2	0.2
		-32.0	1.4	-5.9	-19.6	-7.9	4.0	-31.0	0.1	0.3	0.1	-1.2

Source: Salomon Smith Barney.

In the calculation of partial durations relative to the 5-by-10 swaption volatility, we raise the latter by 1% and raise the longer-dated swaption volatilities (7x10, 10x10 and 20x10) by the initial proportions. It is interesting to note that the POs have higher volatility duration, and the IOs can actually have negative volatility duration, in MOATS. The reason is that, other things being equal, in an environment of higher market volatilities, investors would demand a higher mortgage coupon in order to maintain the same OAS. The higher mortgage rate would result in lower prepayment. For this reason, IOs are not good vehicles to sell market volatilities, even as they remain good vehicles to sell convexity, or equivalently to sell future realized volatilities.

Past Performance

We used MOATS to do a monthly conditional prediction of the mortgage rate. For every month since January 1988, we calculated the OAS for the current coupon and used it, along with next month's yield curve and volatilities, to calculate the new current coupon. We compared this predicted value to the actual level. The error would translate, for example, to a residual price error for a portfolio of IOs well hedged with interest rate and volatility benchmarks. The root-mean-square error is tabulated in Figure 7. We see that there is significant improvement over the current method of using constant spread to the Treasury. For comparison, using a constant spread over the five- or ten-year swap rate also leads to improvement over the choice of Treasury as reference.

Figure 7. RMS Error of Monthly Conditional Prediction of Mortgage Rate

Period	Const Sprd to 10-yr Treasury	Const Sprd to 10-yr Swap	Const Sprd to 5-yr Swap	MOATS
Jan 1988 – Jan 1999	9.2bp	8.8bp	10.9bp	8.3bp
Jan 1999 – Jul 2001	14.2	7.5	6.8	5.2

Summary and Conclusions

We have a new method that projects mortgage rates as an integral part of the calibrated term structure model, using a no-arbitrage approach. The impact on mortgage OAS varies, depending on the yield-curve and volatility environment.

There is little change in relative value ranking.

One important consequence is in the risk management of mortgage portfolios, as the key-rate durations are significantly different.

Appendix

Description of the MOATS method

Notations:

PPM Salomon Smith Barney Prepayment Model

PPM* Simplified Salomon Smith Barney Path-independent Prepayment Model

cc Array of mortgage current coupons, one at each future node in the

Salomon Smith Barney term-structure model. Thus cc_n is the current coupon at the n-th yield curve node in our term-structure model grid, cc_0 being the initial level

Cpn coupon of 30-year conventional mortgage

OAS option-adjusted spread

Price(Cpn,PPM,OAS,cc) Monte-Carlo model price of a new 30-year conventional mortgage with coupon Cpn, using option-adjusted spread OAS, assuming the full prepayment model PPM and the array cc of future current coupons. Other than the non-constancy of the array cc, this is our current pricing method

Price*(n,Cpn,PPM*,OAS*,cc) Backward-induction model price, at the n-th yield curve node, of a new 30-year conventional mortgage with coupon Cpn, using OAS*, assuming the simplified Prepayment model PPM* and the array cc of future current coupons.

We impose the following obvious conditions

cc_0 = actual market current coupon

$1 = \text{Price}^*(n, cc_n, \text{PPM}^*, \text{OAS}^*, cc) \quad (n=0,1,2,...)$

$1 = \text{Price}(cc_0, \text{PPM}, \text{OAS}, cc)$

There are exactly as many equations (2+number of yield-curve nodes) as there are unknowns (OAS, OAS* and cc_0, cc_1,cc_2,...). We can solve the equations for these unknowns with some iteration scheme. In particular, we obtain the array of future current coupons.

Now suppose the yield curve has shifted, as for example in a duration calculation. The initial current coupon cc_0 will not necessarily be at the pre-shift market level. However we assume, as we said we would, that OAS is unchanged, even as we allow OAS* to change to some different level on the ground that the relation of the simplified model PPM* to the full model has changed in the new yield-curve environment. Thus we require

$1 = \text{Price}(cc_0, \text{PPM}, \text{OAS}, cc)$

$1 = \text{Price}^*(n, cc_n, \text{PPM}^*, \text{OAS}^*, cc) \quad (n=0,1,2,...)$

Thus the number of equations (1+number of yield-curve nodes) again is the same as the number of unknowns (OAS* and cc_0, cc_1,cc_2,…). We can solve these equations. In particular, we obtain both the new current-coupon cc_0, and the array of future current coupons.

ADDITIONAL INFORMATION AVAILABLE UPON REQUEST