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Hybrid ARM Valuation

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Executive Summary

We discuss a popular structure backed by hybrid ARM collateral and structured with an embedded forward contract. In the past, investors have used various methodologies to arrive at consensus pricing for hybrid collateral. We discuss the concept of forward price to extend some of these methods to a more robust framework for valuation of the components of the structure, especially the cash flows after the reset. The framework can be further extended to help formulate strategies and price other derivative transactions (swaps and options).

I. Hybrid ARM Valuation — Background

Investors have historically used many different methodologies to value hybrid ARMs.[1] Recently, a spate of structured hybrid ARM issuance, mostly backed by jumbo-A paper, has raised questions on ways to evaluate such structures. Some of these structures separate the cash flows after the first reset as a tranche. Hence, the question of valuing the back end (the adjustable part of the security remaining) after the first reset[2] and the front end separately has been especially critical in evaluating whatever method one may choose to follow. Pertinent concerns include the following:

➤ In the past, investors have broken hybrid cash flows into the fixed part (front end or cash flows until the first reset) and the adjustable part (back end or cash flows after the first reset).[3] It has been a common practice to price the front-end to a balloon put[4] and assume the factor of the back end would be low enough not to affect the valuation significantly. However, given the available factor data from agency hybrid pools, these assumptions for valuing the front end look conservative.[5]

➤ The dollar price of the back end clearly depends on the yield curve and spreads at the first reset. Data on traded hybrids at the first reset are sparse at best, and thus, it is difficult to arrive at reliable assumptions on the price.

Recently, however, as the focus has shifted from pricing the front end to pricing the back end, the main risks of the back end — the uncertainty of the remaining balance and uncertainty of the price at the first reset — largely remain unresolved. In this paper, we try to understand the value of the back end *implied* by the market price of the entire collateral, based on a reasonable prepayment model and the Salomon Smith Barney interest rate term-structure model. We also look at how various modeling assumptions affect the value of the back end and, in turn, the front-end pricing. We hope these discussions will provide investors with a variety of tools to negotiate a better price for such structures.

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[1] *Hybrid ARMs*, Salomon Smith Barney, May 2000, gives an introduction to agency hybrids. Essentially, hybrid ARMs have a fixed and an adjustable part. For example, a 5x1 hybrid would have a fixed coupon for the first five years and floating coupons thereafter with an annual reset schedule.

[2] For purposes of this article, the first reset refers to the weighted-average first reset. For example, the first reset for a new 5x1 would be close to 60 months.

[3] We follow this definition of front-end and back-end cash flows throughout the paper.

[4] This assumes the entire hybrid pays off at the first reset at an agreed-upon price (generally par or slightly higher than par).

[5] We discuss these issues in *Hybrid ARMs*.

II. Hybrid ARM With a Forward Agreement — One of Many Variations

We describe in detail one of the many popular hybrid structures to accomplish the following:

- ➤ Highlight the flexibility now available in the hybrid ARM market, which allows an investor to focus on the cash flows for which they are comfortable managing the risks.
- ➤ Introduce the concept of *forward price* and distinguish it from *horizon* or *scenario prices* in a way that would allow an investor to price any similar structure with a call option, a put option, or a forward agreement.

The Structure

The cash flows from the hybrid collateral are separated into the *front-end* and *back-end cash flows*. The front-end buyer agrees to give up the cash flows on the remaining balance at the first reset either back to the issuer, the underwriter, or a third party at an agreed-upon price (contract price). In return for taking the risk of buying the *unknown* remaining balance at a future date, the back-end buyer[6] is compensated when the contract is closed (on the current settle).[7] The predetermined contract price can theoretically be at a level that all parties agree on. In practice, however, it is usually par. We show later that there is a fair compensation (or premium) for every contract price, but for convenience we will use a contract price of par in our analyses.

An Analytic Description

For purposes of future reference, we outline the naming convention for the various cash-flow components. Please note that the word *value* in this context can be interpreted as the present value for convenience, but would technically hold true for value at any point in time under a consistent set of assumptions.

Let, CF(1), CF(2) . . . CF(k), CF(k+1) . . . CF(n) denote the monthly cash flows for a hybrid with n months to maturity and k months to the first reset. Also let B(1), B(2) . . . B(k), and B(k+1) . . . B(n) denote the remaining balances at the beginning of the respective months.

H is the value of the hybrid collateral or the full cash flow — CF(1) . . . CF(n).[8]
FE is the value of the cash flows till the first reset (front end) — CF(1) . . . CF(k).
BE is the value of the cash flows after the first reset (back end, corresponding to the remaining balance at the first reset) — CF(k+1) . . . CF(n).[9]

[6] We follow these definitions of the front-end buyer and the back-end buyer throughout the paper.

[7] Convention for dates: Current Settle or Settle or Today (in our case, July 13, 2001) -- the date on which the contract is closed, the front-end buyer pays the cost of the front end and the back-end buyer collects the compensation (premium). Future Settle or Forward Settle or Horizon (in our case, May 13, 2006, the first reset at month 58) — the date on which the front-end buyer hands over the remaining balance to the back-end buyer at the contract price, in this case par.

[8] Use SA1.FE A1 on Yield Book™ without the call option.

[9] Use SA1.FE A2 on Yield Book™.

FWD is the value of the remaining principal at the first reset at the contract price (in this case, par). In this case, the value of B(k+1)*100.

F is the front-end forward (front-end cash flows with the forward contract) — FE + FWD, because the front-end investor gets a cash inflow at the beginning of month k+1.[10]

B is the back-end forward (back-end cash flows with the forward contract) — BE - FWD, because the back-end investor experiences a cash outflow at the beginning of month k+1.[11]

Hence,

$$H = FE + BE$$
$$= (FE + FWD) + (BE - FWD)$$
$$= F + B$$

Thus, the front-end buyer has exposure to F, which is essentially the front-end cash flows plus the value of the remaining principal at par at the first reset. The back-end buyer has exposure to B, which is the back-end cash flow with a liability of buying the remaining balance at par at the first reset.

In Figure 1, we use the following 5x1 hybrid for further analysis.

Figure 1. 5x1 Hybrid Characteristics, as of 13 Jul 01

Type	Index	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll	Price	OAS to Swap (bp)	Duration	Convexity
5x1[a]	One-Year CMT	6.34	7.08	250	275	2	58	100.64	25	2.2	-1.6

[a] Modeled as SA1.FE A1on Yield Book™. It can be run to call at the first reset. It prices H when run without call and prices F when run with the call option. We use a modified hybrid prepayment model for these valuations. We modified our FNMA 5x1 hybrid prepayment model to capture higher level of refinancings for higher loan sizes and higher current coupons for jumbos.
Source: Salomon Smith Barney.

Common Pricing Assumptions

Traditionally, in pricing the back end, BE will be priced under a set of prepayment, yield curve, and spread assumptions. The pricing might be done under the assumption of a constant CPR and a discount margin or under the assumption of an OAS in combination with a prepayment model. As an illustration, we price the back end of the 5x1 hybrid at constant OAS and an unchanged yield-curve scenario at the first reset.

Figure 2. 5x1 Hybrid Characteristics at the First Reset, With 13 Jul 01 Yield Curve

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll	Remaining Balance	Price	OAS to Swap (bp)	Duration	Convexity
5x1[a]	6.03[b]	6.28	250	275	60	12	0.19	101.75	25	0.9	-0.6

[a] Net coupon at first reset = Unchanged one-year CMT + net margin = 3.53% + 2.50% = 6.03%. [b] Gross WAC at first reset = Unchanged one-year CMT + gross margin = 3.53% + 2.75% = 6.28%.
Source: Salomon Smith Barney.

[10] Use SA1.FE A1 on Yield Book™ with the call option.

[11] As shown subsequently we would think of B = H - F.

Figure 3. 5x1 Hybrid Characteristics at the First Reset, With 13 Jul 01 Yield Curve Modified for Different Scenarios

Type	Coupon (%)	WAC (%)	Age (Months)	Months to Roll	Scenario	-300bp	-200bp	-100bp	0bp	+100bp	+300bp	+500bp
5x1[a]	6.03	6.28	60	12	Horizon Price	101.76	102.06	101.87	101.75	101.20	99.94	95.37
					Remaining Balance	0.004	0.01	0.09	0.19	0.21	0.25	0.31

[a] We use 500bp as an upward shift because the yield curve is steep and the implied volatility is high. We pick the two-year LIBOR rate as an example, because the hybrid is closest to the two-year Treasury in duration. The two-year forward is about 250bp higher than today's rate. The volatility is about 20% annualized. At an average level of 5%, the standard deviation over five years would be 5%*20%*sqrt(5) = 225bp. Even at one standard deviation the upward shift would be 250 + 225 = 475bp. On the other hand, the one standard deviation downward shift would be 250bp - 225bp = 25bp, a positive shift. Overall, these numbers indicate that in an arbitrage-free environment, over the next five years the rates are biased to shift upward.
Source: Salomon Smith Barney.

The implicit assumption in this analysis is that the yield curve is unchanged between July 13, 2001 (the current settle), and May 13, 2006 (58-month horizon). Hence, the price of 101.75 represents one of the many possible horizon prices. We can similarly calculate a set of horizon or scenario prices[12] at the first reset under different scenarios.

This analysis gives an idea of sensitivity of the scenario-horizon prices under different rate shifts, but still does not help us arrive at the fair price of the back end if the contract price is decided today. Also, this analysis gives us the price for a bond of $100 face value at the first reset. As we will see in the next section, the price we need to arrive at needs to incorporate the distribution of remaining balances under these scenarios, since the transaction on the forward settle will take place *only* on the remaining balance.

[12] Hence, the horizon or scenario price denotes the price of the back end ($100 face-value) some assumed yield curve and spread at the horizon or the first reset and a single realized interest rate scenario between the current settle and the horizon.

III. Valuation and Interest Rate Sensitivity

In this section we describe the notion of forward price, which provides us with a benchmark for the fair price for the back end. A number of assumptions are involved in calculating this price. We discuss the impact of these assumptions later in this report.

Forward Price at the First Reset

Theoretically, the back-end buyer and the front-end buyer can enter into a contractual agreement on the current settle to transact the remaining balance at the forward price, as implied by the price of the collateral, at *no compensation* to either. This forward price is such that the front-end buyer agrees to sell the remaining balance and the back-end buyer agrees to buy the remaining balance at that price on the first reset (or the forward settle date) without any cost to either.

The total *market* value of the transaction at the first reset, however, has two components, both of which are unknown today: the remaining balance and the market price. We describe the methodology for calculating the forward price in Appendix A.

Intuitively, the *forward price* for the collateral can be thought of in several different ways.

- The forward price is the *weighted average* of *possible market prices* at the first reset (the possibilities arise out of the family of arbitrage-free interest rate scenarios).[13] The weights can be thought of mainly as the remaining balances (because, in our example, the discounting factors between the current settle and forward settle, which also form a part of the weights, have a lesser impact on the weights than the remaining balances).[14]
- The forward price is also that *unique* price such that if the hybrid were to be mandatorily called at that price on the first reset under *each and every one* of the arbitrage-free interest rate scenarios,[15] it would not change the dollar price of the entire hybrid today.

These two descriptions are equivalent. The hybrid at the current settle is worth the front-end plus the back-end cash flows. Of the two parts, at the current settle, the back-end cash flows are worth the average of the discounted values of all the possible scenario market values at the first reset. Because the forward price captures the average of these market values, the value of the back end would remain unchanged *irrespective* of whether the averaging is done with all the possible

[13] We will refer to the balance and price at the first reset for each of these arbitrage-free scenarios as the horizon/scenario balance and horizon/scenario price.

[14] Essentially the weights are the product of the discounting factors (between the current settle and the forward settle) and the remaining balances for every interest rate path. The component with higher variability affects the variability of the weight more than the other.

[15] For purposes of valuation, the call price in this paper refers to a mandatory call of the bond at the forward settle for every arbitrage-fee interest rate scenario. When we refer to a call option we explicitly state it.

scenario prices or it is done with just one price (the forward price). When the forward price is used to compute the market value of the back end for every scenario (instead of using the respective scenario prices), the value of the back end and, in turn, the value of the hybrid does not change. Hence, the price of the hybrid collateral *implies* a mandatory embedded call for the remaining balance at the forward price at the forward settle (in our case, the first reset). This shows the equivalence of the weighted average and the implied call concepts.

Figure 4 shows the forward price for the collateral at the first reset is 97.58. As opposed to one horizon/scenario price, which is calculated under the assumption of an arbitrary yield-curve scenario, here the forward price is an average of many such horizon/scenario prices under arbitrage-free interest rate scenarios.

Figure 4. 5x1 Hybrid Forward Price at the First Reset, With 13 Jul 01 Yield Curve

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll (Months)	OAS to Swaps (bp)	Current Settle Price (H) 13 Jul 01	Forward Price on 13 May 06
5x1	6.34	7.08	250	275	2	58	25	100.64	97.58

Source: Salomon Smith Barney.

Figure 5 shows a chart of prices with the *product of discounting factors* and the *remaining balances* as the frequency weights captured by the arbitrage-fee rate scenarios. We draw attention to the fact that one of the reasons the forward price is well below par is the high remaining balances during a selloff. The high weights next to the low prices are explained by the chances[16] of a heavy selloff over the next five years in a steep yield-curve and a high-volatility situation.

Figure 5. Distribution of Scenario Prices[a] at the Forward Settle (First Reset 13 May 06) Under Arbitrage-Free Scenarios

Dollar Price	Normalized Weights
82.14	0.10
95.60	0.14
98.38	0.12
99.36	0.15
100.13	0.23
100.76	0.12
101.55	0.09
102.69	0.05

[a] These prices are midpoints of price buckets for which we calculate the weights.
Source: Salomon Smith Barney.

[16] “Chances” refers to probabilities in the risk-neutral framework. As mentioned, all valuations in this paper have been carried out using the Salomon Smith Barney Term Structure model, which uses the risk-neutral framework. This framework comprises a family of arbitrage-free interest-rate paths for which on average the forward rates are realized. In this framework, the forward rates are rising in a steep yield-curve environment, increasing the chances of a selloff. However, there is an equivalence between the risk-neutral framework and probability assumptions in any other framework. For example, in a steep yield-curve environment, investors might either be risk-neutral (assume steep forwards are realized and use risk-free discounting rates) or they might subscribe to any other set of probabilities for future interest rates (and discount by a higher rate of expected return). But in either case they will derive the same valuations for traded securities. The risk-neutral framework is, however, the accepted framework for calculation of forward prices or any other derivative price.

The Actual Contract and the Premium (Compensation for the Back-End Buyer)

Most contracts are struck at par. The reasons for choosing a price of 100 instead of the actual forward price might be one of the following:

- A price of par leads to a standard price that buyers and sellers can use for various transactions.
- A price of par is equivalent to a balloon payment for the buyer of the front-end forward (F) and allows him to use pricing methods similar to par-to-put.[17]

But when the contract price is different from the theoretical forward price, the back-end buyer or front-end buyer needs to be *compensated* based on whether the forward price is lower than or higher than par. The compensation is the premium for the contract.

The Premium

Next, we calculate the premium for a contract struck at par. We reiterate that as shown in Figure 6 — the implied call price (or the forward price) for the hybrid collateral is 97.58.[18] However, the contract price is 100, or 2.42 above the forward price of 97.58. Hence, we would expect the price of the collateral to increase when we *force* a price of par on the remaining balance at the first reset. However, that is precisely what the front-end investor is paying for — the guarantee of a price of par, a price that is higher than the forward price implied by the current market value of the collateral. He is thus expected to pay more than the collateral price. In Figure 6, we show that the price of the hybrid increases 0.26 to 100.90 when called at par at the first reset. In other words, the forward price of 97.58 is the implied call price for the collateral priced at 100.64. However, the collateral called at par is equivalent to the front-end forward and is priced at 100.90.

To the front-end buyer, the front-end forward is worth 0.26 more than the hybrid collateral cash flows, because the back-end buyer is paying 2.42 over and above the forward price. Hence, this 0.26 is also the premium the back-end buyer should receive. As explained in Appendix B, the premium is also the differential between the contract price and the forward price multiplied by the average of the product of the remaining balances and discounting factors across the scenarios.[19]

The premium increases as the forward price falls below the contract price. The premium also increases with the increasing remaining balance when the forward price is below the contract price — because the back-end buyer is paying more on a higher balance and should be compensated for both.[20]

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[17] This is currently a popular way of valuing a hybrid — by assuming the entire bond will pay down at the first reset.

[18] SA1.FE A1, when called at 97.58 at the first reset or the theoretical forward price, will be priced at 100.64 today (which is unchanged from the original valuation).

[19] The average of the remaining balances is 0.16 and the average of the discounting factors is 0.68. The premium 0.26 is approximately 2.42*0.16*0.68. This is a simplification, because the average of products is not mathematically the same as product of averages. Nevertheless, it provides the intuition of the premium being the present value of the price differential applied on the remaining balance.

[20] It is possible that for a high-premium hybrid or in an inverted yield-curve scenario, the forward is *above* the contract price of par. In such a case it can be argued that the back-end buyer should pay to get into the contract. However, given that the perceived risk on the back end is high, it may not be realistic.

Figure 6. 5x1 Hybrid Front-End Forward Price and the Back-End Premium, With 13 Jul 01 Yield Curve

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll (Months)	OAS to Swaps (bp)	Current Settle Price (H) 13 Jul 01	Forward Price on 13 May 06	Current Settle Price When Called at Par (F)	Premium
5x1	6.34	7.08	250	275	2	58	25	100.64	97.58	100.90	0.26

Source: Salomon Smith Barney.

The Front-End Forward and the Back-End Forward — The Two Components

The hybrid cash flows can be expressed as the sum of present values of the front-end forward and the back-end forward or H = F + B, or 100.64 = 100.90 + (-0.26).

The hybrid cash flows have been split up into two components. The first component is the front-end cash flow along with an agreement to sell the remaining balance at par at the first reset and is priced higher than the collateral because the par price is greater than the fair forward price. The second component is the back-end cash flow, which is a combination of a liability to pay par for the remaining balance and an asset of the back-end cash flow. The present value of the second component is negative, because the back-end buyer has agreed to pay more than the fair forward price as of today and he is compensated for it today with a premium that is equal to the negative present value. The premium received by the back-end buyer is the negative of the present value of the back-end forward and, hence, is a positive quantity.

Clearly, we can compute premium for any contract price as shown in Appendix B. We used par because it is the most common contract price used.

Interest Rate Sensitivity — Duration and Convexity of the Different Parts of the Structure

Figure 7 shows the duration and convexity of the hybrid cash flows and the front-end forward. (Please see Section II for a recap of what we mean by front-end forward.) The front-end forward has lower duration and better convexity.

Intuitively, the explanation is as follows. The duration and convexity of the two securities — H (hybrid collateral) and F (front-end forward) — are attributable to the interest rate sensitivity of the front-end cash flows and the interest rate sensitivity of the back-end cash flows. The rate sensitivity of the back end has two components, the rate sensitivity of the remaining balance and the rate sensitivity of the back-end scenario-horizon prices. The difference between the two securities (H and F) is that for the hybrid collateral, the back-end sensitivity is a result of the distribution of balances *and* prices, whereas for the front-end forward, the back-end sensitivity is a result of the distribution of remaining balances *only* (the price is fixed at the contracted price of par). Hence, the back-end *price* sensitivity to interest rates, which has a positive duration and a negative convexity like most mortgage-backed securities, is *absent* in the front-end forward, reducing duration and improving convexity. (See Appendix C for further calculations.)

Figure 7. Duration and Convexity of the 5x1 Hybrid and Front-End Forward, With 13 Jul 01 Yield Curve

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll (Months)	Current Settle Price 13 Jul 01	OAS to Swap (bp)	Duration	Convexity
5x1 H	6.34	7.08	250	275	2	58	100.64	25	2.26	-1.68
5x1 F	6.34	7.08	250	275	2	58	100.90	25	1.92	-1.41

Source: Salomon Smith Barney.

Now we show the sensitivity of the back-end forward. Because the market value of this component is very small (and can be zero if the contract is struck at the theoretical forward price), we compute the change in dollar price resulting from the change in interest rate rather than the percentage change in price. So, rather than using duration and convexity, we use the change in dollar price *resulting from* duration and convexity as a measure of interest rate risk.

We derive the dollar price change per 1bp shift in interest rate resulting from duration and convexity for H and F.[21] Then we derive the changes for B by simply subtracting the changes for F from those of H. The back-end forward has positive duration and negative convexity characteristics. It is not surprising that because the front-end forward has shorter duration and better convexity than the collateral cash flows, the back-end forward will be *left* with positive duration and negative convexity.

Thus, after splitting up the full cash flows of the hybrid we simply wind up with two positive duration and negative convexity bonds. Because the contract price is higher than the theoretical forward in our current example, however, the back-end forward — B — is similar to a liability cash flow with positive duration (liability decreases with the fall in interest rates).

Interested investors can separately examine a finer calculation duration and convexity of the cash flows and the forward contract, although we do not think it is critical to understanding the structure (see Appendix D).

Figure 8. Duration and Convexity of the 5x1 Hybrid, the Front-End Forward and the Back-End Forward, With 13 Jul 01 Yield Curve

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll (Months)	Current Settle Price 13Jul 01	OAS to Swap (bp)	DV01	Dollar Price Change per bp due to CNVX(10^-5)
5x1 H	6.34	7.08	250	275	2	58	100.64	25	0.0224	-8.2
5x1 F	6.34	7.08	250	275	2	58	100.90	25	0.0190	-6.9
5x1 B	6.34	7.08	250	275	2	58	-0.26	25	0.0034	-1.3

Source: Salomon Smith Barney.

[21] For example, duration and convexity of H are 2.257 and –1.678 for a price of 100.926 (100.64 flat price + 0.28 accrued interest). Hence DV01 is 100.926*2.257*0.0001.

Similarly, the effect resulting from convexity for a 1bp shift would be 100.926*(-1.678)*(0.0001^2)*0.5/10^-2 = -8.2*(10^-5).

IV. Impact of Valuation Assumptions and Strategy Implications

We begin this section with a pair of questions. If the hybrid is at a significant premium today, should the back-end buyer expect a compensation for agreeing to buy the back end at par? Or, for example, if the hybrid is at a premium, should the front-end buyer even agree to sell the back end at par?

Clearly, pricing assumptions would affect the calculation of the forward price and the premium. We look at OAS, volatility, and prepayment assumptions and their role in arriving at a fair price. As we will see, it is important to focus on the right set of assumptions in a specific contract.

We start with a discussion of the impact of OAS assumptions, because sensitivity of price to spreads is simpler to tackle given that the cash flows remain unchanged.

OAS Assumptions and Their Impact on Pricing

Impact of OAS on the Drop

We begin with the calculation of the "drop" because it is traditionally used in the context of forward pricing in the roll market.[22] The drop is the difference between the current settle price and the forward price for the collateral, and in our case, it is 100.64 – 97.58 = 3.06.

To arrive at some intuition about the direction and magnitude of this drop, we use various OAS assumptions to calculate the forward price and the drop. To recap, H is the hybrid collateral, with the implied call price the same as the forward price, F is the front-end forward with a call price of par. The premium is the difference between the two. Current settle is July 13, 2001, and the forward settle is May 13, 2006, or the first reset.

Figure 9. Forward Price and Premium Under Different OAS Assumptions, With 13 Jul 01 Yield Curve

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll (Months)	OAS to Swap (bp)	Current Settle Price for H as of 13 Jul 01	Forward Price as of 13 May 06	Drop	Current Settle Price for F (When Called at Par) as of 13 Jul 01	Premium for a Contract at Par
5X1	6.34	7.08	250	275	2	58	-100	103.44	100.00	3.44	103.44	0.00
							10	100.97	97.79	3.18	101.20	0.23
							15	100.85	97.70	3.15	101.10	0.24
							25	100.64	97.58	3.06	100.90	0.26
							75	99.57	96.80	2.77	99.91	0.34
							700	87.89	88.66	-0.71	88.88	0.99

Source: Salomon Smith Barney.

22 Forward pricing in the roll market is given by P = d1*C + d2*(f*F), where P is the current settle price, f is factor, and F is the forward price, and d1 and d2 are the respective discount factors. See *Gaining Exposure to Mortgage Benchmarks*, Salomon Smith Barney, May 2001. Forward price in the roll market is calculated with the assumption of a short rate. In our case, forward price is similarly calculated based on expected values across a family of arbitrage-free interest rates, as explained in Appendix A. The horizons in the roll markets are short, making volatility of rates a less critical factor.

Why should the drop decrease with increasing OAS? In short, the spread duration of a shorter bond will be smaller. The sensitivity of forward price to OAS is captured by the sensitivity of the price of a five-year-old bond.[23] Hence, the forward price does not decrease as much as the current settle price does with increasing OAS. Another way to look at it is: because the cash flows are unchanged, a higher required return (or a higher OAS) would increase the scenario-horizon prices[24] relative to the current price — since that is the only way to increase the returns between the current settle and forward settle. This would lead to an increase in the forward price (a weighted average of the scenario-horizon prices) relative to the settle price and a decrease in the drop.

Impact of OAS on the Premium (Compensation to the Back-End Buyer)

We will begin to appreciate the full impact of this analysis as we compute the premium for the back end at the different OASs. To recap, the premium measures how much more the front-end forward is worth than the collateral *because* the back-end buyer is agreeing to pay a higher contract price relative to the forward price. Thus, the premium increases as the forward price falls below the contract price with higher OASs, as seen in Figure 9.

The collateral price is what is left *after* the back-end buyer is compensated from the price that the front-end buyer pays. With increasing OASs, the settle price and the forward price decrease and the premium increases. In other words, with increasing OASs, the front-end buyer pays a lower dollar price and the back-end buyer receives higher premiums — so *both* benefit at a cost to the originator.

Furthermore, the premium increases[25] with increasing OASs, but not as much as the settle price decreases. For example, between 15bp and 25bp the settle price for the collateral decreases 0.21 (100.85 - 100.64), and the front-end forward decreases 0.19 (101.09 - 100.90), while the premium increases barely 0.02. The reasons are twofold: with increasing OASs, the forward price does not decrease as much as the settle price (or, the drop decreases) *and* the premium is calculated only on the remaining balance.

The Tradeoff Between Front-End and Back-End Buyers

However, there is no guarantee that both front-end and back-end buyers would buy at the same OAS. For example, in an extreme case, if the front-end buyer pays an extremely high premium price of 103.44, the back-end buyer is theoretically entitled to no premium (because the forward price is exactly equal to par). But if he can

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[23] Forward price in our calculation is dependent on remaining balance, but we are looking at the impact of OAS, which does not change the remaining balance in anyway. Hence, spread duration can be used to judge the impact.

[24] In an oversimplified case if P = C/(1+r) + C/(1+r)^2 + C/(1+r)^3, where P is the settle price, C is the cash flow and Z = C/(1+r) is the horizon price at time = 2. d(Z-P)/dr = C[2/(1+r)^3 + 3/(1+r)^4]>0, or the drop, which is (P – Z), will decrease with an increase in r. Also spread-duration of P is higher than that of Z.

[25] Recall that premium received is the negative of the quantity $(P_f - 100)E(B_i D_i)$ and increases as higher OASs decrease P_f , the forward price, or the weighted average of the back-end scenario prices.

negotiate a premium of 0.23, he has effectively bought the back end at an OAS of 10bp, or 110bp better than the front-end buyer.

Another more realistic example would be the following. Let us assume the entire structure is priced at 15bp. The collateral is priced at 100.85, but the front end sells at 0.10 higher than fair value or at 101.20 = 101.10 + 0.10. The back-end buyer, who is the beneficiary, receives the extra premium of 0.10 and receives a total premium of 0.24 + 0.10 = 0.34, which is worth almost 75bp OAS to him. A 5bp (15bp - 10bp) cost in OAS to the front-end buyer is a benefit of 60bp (75bp - 15bp) to the back-end buyer.[26] In other words, if the value of collateral were kept constant at 101.85, a 5bp tightening on the front-end forward would mean a 60bp widening on the back-end forward.

So the answer to the first question at the beginning of the section is yes, the back-end buyer may be eligible for a premium even if the bond is priced above par[27] (as is true when all the components are valued at 15bp OAS), or he can negotiate a pricing OAS to obtain a premium higher than the theoretical value (as shown in the example).

Volatility Assumptions and Shape of the Curve

High volatility assumptions in a steep yield-curve environment (as in the current environment) can lead to steep forward rates[28] and a *low* forward price, resulting in substantial compensation to the back-end buyer (see implied volatility calculations in Figure 10). However, the compensation would decrease with decreasing volatility assumptions. To illustrate the point we price the bonds with historical volatilities, which are significantly lower than current implied vols[29] (see fixed-historical volatility calculations in Figure 10). The settle price and the forward prices are higher with the lower set of volatilities. Furthermore, because any increase in forward prices would decrease the premium (compensation), we see a decrease in the premium with lower volatilities.

Hence, a lower volatility assumption in a relatively steep yield-curve environment hurts the front-end and the back-end buyers to the advantage of the originator. Conversely, a higher volatility assumption would help the buyers obtain a better price.

[26] Steps: Value of Collateral at 15bp OAS is 100.85 = FE + BE = FE + FWD @ 97.70 = (FE + FWD @100) + (FWD @97.70 – FWD @ 100) = (FE + FWD @ 100) + (BE- FWD @ 100) = 101.20 + (-0.24). All components valued at 15bp OAS. However, 100.85 = 101.20 + (-0.34) = (FE + FWD @ 100) valued at 10bp OAS + (BE- FWD @ 100) valued at 75bp OAS. It might appear unreasonable, but the small additional premium gives the back-end buyer the leverage to weather a drop of almost a point in market price on average. We will show some relevant calculations in Chapter IV.

[27] A high coupon hybrid or flatter yield curve can certainly price the full cash flow at a high premium.

[28] In the risk-neutral framework

[29] For example, the current ten-year cap volatility is about 18.5% (as of July 13, 2001) whereas the historical ten-year volatility used in the Salomon Smith Barney models is about 14%. The lower maturity volatilities are even more divergent.

Figure 10. Settle Prices and Premium, with 13 Jul 01 Yield Curve at 25 OAS to Swaps

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Mos.)	Months to Roll	Volatility	Current Settle Price 13 Jul 01 H	Settle Price as of 13Jul 01 F	Premium for B	Forward Price as of 13 May 06
5x1	6.34%	7.08%	250bp	275bp	2	58	Implied	100.64	100.90	0.26	97.58
							Fixed Hist	101.04	101.19	0.15	97.96

Source: Salomon Smith Barney.

However, much of the advantage of higher volatility assumptions to the buyers is lost when the curve is relatively flat as seen in Figure 11. (For example, the compensation remains relatively unchanged for both implied and historical volatility assumptions.)

Figure 11. Settle Prices and Premium, With Flat 4.5% Treasury and 5% Swap Curve at 25 OAS to Swap, With Implied Vol 13 Jul 01

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll (Months)	Volatility	Current Settle Price 13 Jul 01 H	Settle Price as of 13 Jul 01 F	Compensation for B	Forward Price as of 13 May 06
5x1	6.34	7.08	250	275	2	58	Implied	100.56	100.58	0.02	99.99
							Fixed Hist	100.77	100.78	0.01	99.99

Source: Salomon Smith Barney.

The answer to the second question at the beginning of the section is also a yes. If the curve is relatively steep and rate volatilities are relatively high, the forward price will be below par, indicating a potentially low horizon price at the first reset.[30] Hence, a front-end buyer might find it attractive to get a forward agreement to sell the remaining balance at par, even if the bond is at a premium now.[31]

Prepay Assumptions

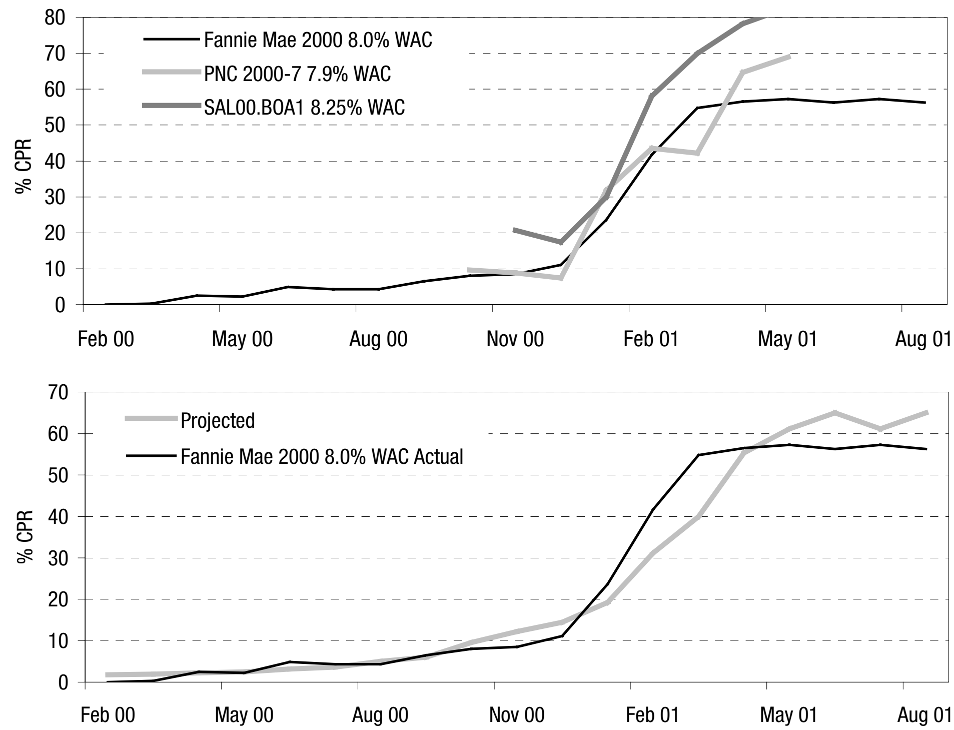
In Figure 12, we attach two prepayment graphs. The first one is a comparison of some recent jumbo hybrid speeds with agency hybrid speeds, and the second is a comparison of speeds projected by the Salomon Smith Barney prepayment model versus actual agency hybrid speeds. Even though the model projections are reasonably close to actual agency speeds, the jumbo hybrids have prepaid faster than agencies during the refi wave (and a little slower at the beginning). Accordingly, we have used a modified agency hybrid model to price the structure (mainly modified to account for higher refinancing propensities for larger loan-size jumbos and higher current coupons for jumbo loans).

However, with the limited data on jumbo hybrids, there is uncertainty about speeds in the future. We show an analysis of settle and forward prices under different prepayment assumptions in Figure 13.

[30] Again in a risk-neutral framework. However, as mentioned, an investor might either believe that forwards are likely to be realized and believe the horizon prices will be low and use risk-free discounting rates or he might have other probability assumptions about future rates but would use higher discounting rates (because expected returns will be higher than risk-free rates in a steep yield-curve environment). Because all valuations use present-value of discounted values, these methods are equivalent – in other words, we will derive the same value for the premium (compensation) and the same price for the collateral in both frameworks.

[31] We would have obtained similar results if the bond were priced at -5 OAS or 101.30, for example.

Figure 12. Prepayments of Hybrids — Agencies and Jumbos



Source: Salomon Smith Barney.

Figure 13. Settle Prices and Premiums Under Various Prepayment Scenarios, 13 Jul 01

Type	Coupon (%)	WAC (%)	Net Margin (bp)	Gross Margin (bp)	Age (Months)	Months to Roll (Months)	Percentage of Refinancing (%)	Current Settle Price 13 Jul 01 H	Settle Price as of 13 Jul 01 F	Premium for B	Forward Price as of 13 May 06
5x1	6.34	7.08	250	275	2	58	75	100.74	101.00	0.26	98.24
							100	100.64	100.90	0.26	97.58
							150	100.57	100.78	0.21	96.30

Source: Salomon Smith Barney.

A higher prepay assumption reduces the price for the front-end buyer, but it may not benefit the back-end buyer. The forward price decreases with increasing prepays, which is supposed to help the premium. However, the smaller remaining balance actually hurts the premium. In other words, since the back-end buyer is essentially picking up the extension risk, he needs to be compensated less when the prepays are expected to be fast.

Strategy for Buyers

We discussed the characteristics of the various components of the deal. We can use understanding of the pricing mechanism to help investors meet their investment objectives.

Front-End Buyer

The front-end buyer would most likely have the investment objectives of a hybrid ARM buyer who looks for good carry and slow prepay speeds compared to one-year

ARMs, but lower duration compared to fixed rates. However, with the forward agreement he has several advantages over a hybrid collateral buyer.

Since he knows the final maturity of the cash flows, it is easier for him to manage the cash flows. For example, he could invest in several front-end forwards with staggered maturities to match his liability cash flows.

The structure also makes the front-end forward more amenable to comparison to PACs or other structured CMOs with defined maturities.

The forward contract feature helps eliminate the extension risk at the first reset and the front-end investor could also hedge against shortening risk with POs. Hence, it is possible to create tight WAL structures, in all probability cheap to the CMOs.

Clearly, the front-end agreement helps hedge against extension risk in case of a selloff and against price risk in case of a LIBOR-CMT spread widening, when the front-end buyer has no other mechanism to hedge against those risks.

Back-End Buyer

The main risk the back-end buyer is compensated for is that in a heavy selloff the horizon price of the back end will be below par and the remaining balance will be significantly higher than expected. Essentially, the back-end buyer gets paid a higher premium as the chances of extension become higher than the chances of shortening.

For the sake of argument, we assume the compensation in Figure 6 grows at a risk-free rate for five years to 0.33 (0.26*1.05^5). If P and B are horizon price and the remaining balance of the back end, respectively, the back-end buyer would take a loss if $PB+0.33 < 100B$ or $P < 100-0.33/B$. If we assume B is 0.2, the back-end buyer takes a loss when $P < 98.35$. Also, for every additional 0.10 in future value of premium, the back-end buyer can handle a price drop of 0.1/0.2 or 0.5 without taking a loss. In the price distribution in Figure 5, only about 25% of the cases have a lower price than 98.35.[32] Even in the scenario analysis in Figure 3, we found that only in a 500bp selloff was the horizon price lower and the remaining balance higher than the numbers here. However, since the compensation is the fair price, the back-end buyer does bear a disproportionate risk in a heavy selloff, when the prices are extremely low and the balances are extremely high.

The back-end buyer would typically be an investor who wants to gain exposure to the slow prepays of a seasoned floater without taking any shortening risk on the front-end prepays. (In the case of heavy prepays during a rally, either the deal pays off with the back-end buyer pocketing the premium, or the rally leads to premium horizon prices, in which case the back-end buyer makes a profit.)

Hence, buying the back end is a good strategy for an investor if he has no mechanism for hedging against the front-end prepays or, on the other hand, he is hedged against an extreme selloff or a LIBOR-CMT widening with out-of-the money caps (or other short LIBOR products).

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[32] In the risk-neutral framework.

Other Comments

With structured hybrids, the number of options for the investor essentially increases. The front-end buyer can take advantage of the nice structured cash flow, and the back-end buyer gets exposure to a short-duration seasoned collateral that would prepay slowly after reset and gets paid to do it. The exposition of forward price is important for any structure around hybrid ARMs. We have calculated prices for call and put options on the back end — which are also hinged on the concept of forward price. Any other transaction involving the front end or the back end — for example, swapping the front end to a floating instrument or swapping the back end to a fixed-instrument would require understanding of the forward price. In this paper, we have provided an overview of a basic tool to facilitate any derivative transaction on hybrid cash flows.

Appendix A. Calculation of Forward Price

The forward price is a weighted average of horizon market prices. The average is calculated under assumption of risk-neutral arbitrage-fee interest rate paths. Formally defined, it is that price that will equate the expectation of the discounted market value (the product of the market price and the remaining balance) to the expectation of the discounted forward value (product of the forward price and the remaining balance). Clearly, for every interest rate scenario, the market price and remaining balance would be different. The forward price, however, is a constant number given the forward settle date and pricing assumptions (mainly, the prepay and spread assumptions).

All calculations are done at the current settle date;

Let P_f be the forward price at the forward settle;

Let B_i be the remaining balance for BE (the back end) at the first reset for interest rate path i between current settle and first reset;

Let P_i be the market price at the first reset for BE for interest rate path i; and

Let D_i be the discount factor (product of all forward discount factors) for the interest rate path i between the current settle and the forward settle.

Then the following equation holds.

$E[(P_i - P_f)B_iD_i] = 0$, where E is the expectation operator over the interest rate paths. For arbitrage-free risk-neutral paths the expectation is a simple average.

If we denote B_iD_i by w_i, since P_f is contant across i , $P_f = \sum_i P_i w_i / \sum_i w_i$

Because the expectation is equated to zero, a transaction of the back end at the forward price is like exchanging the distribution of P_i (exposure to the uncertain price distribution) for a certain but equivalent P_f, and theoretically should cost nothing. Please note all prices are calculated at a constant OAS with a prepayment model to capture cash flows along every interest rate path. In our calculations we use a modified Salomon Smith Barney hybrid prepayment model version with OAS assumptions that reflect current market prices.

In addition,

$H = FE + BE = FE + E(P_iB_iD_i) = FE + P_f E(B_iD_i) = FE + FWD@P_f$.

Thus, the price of the hybrid implicitly assumes a mandatory call price of P_f at the first-reset or the forward settle.

Appendix B. Calculation of the Premium (Compensation)

$H = FE + BE$

$= FE + FWD\ @\ P_f$ (since BE is the same as a call at the forward price)

$= FE + FWD\ @\ 100 + (BE - FWD\ @\ 100)$

$= FE + 100E(B_i D_i) + (P_f\ \text{-}100)E(B_i D_i)$

= Price of Front-End called at Par + Price of buying the back-end at Par

= Price to the front-end buyer to sell remaining balance at Par +

Premium paid by the back-end buyer for buying the remaining balance at Par

$= F + B$

Hence, *the premium paid* $= H - (FE + 100E(B_i D_i))$ (difference in value of the collateral and the front-end forward) $= (P_f\ \text{-}100)E(B_i D_i)$ (difference of the forward price and the contract price multiplied by the average of the product of the scenario-balances and discount factors)

In our case, $H = F + B$, 100.64 = 100.90 + (-0.26)

Because in this case the premium paid is negative, it is, in reality, premium received or compensation received by the back-end buyer. Thus, the compensation to the back-end buyer is the average difference between the theoretical forward price and the contract price discounted back from the forward settle to the current settle.

Appendix C. Calculation of Interest Rate Sensitivity

Even though it is intuitive to understand that a security with a balloon payment will have shorter duration and better convexity compared to the full cash flows, we lay out a small mathematical proof for the same.

The notations we use are *M* for *market value, P* for *price,* and *B* for *balance.* Hence, *M = PB.* We use the subscripts H for the hybrid collateral cash flows, FE for the front-end cash flows, BE for the back-end cash flows, and F for the front-end forward. Refer to Section II for a description of these components.

Using Taylor series expansion, we can write the change in market value of the hybrid collateral with interest rates as,

$$dM_H = \partial M_H / \partial r \Delta r + 1/2(\partial^2 M_H / \partial r^2 \Delta r^2), or$$

$$dM_H = dM_{FE} + dM_{BE}$$

$$= \quad (\partial M_{FE} / \partial r + \partial M_{BE} / \partial r)\ \Delta r + ½(\partial^2 M_{FE} / \partial r^2 + \partial^2 M_{BE} / \partial r^2)\ \Delta r^2$$

$$=$$

$$(\partial M_{FE} / \partial r + B_{BE} \partial P_{BE} / \partial r + P_{BE} \partial B_{BE} / \partial r)\Delta r +$$
$$1/2(\partial^2 M_{FE} / \partial r^2 + B_{BE} \partial^2 P_{BE} / \partial r^2 + P_{BE} \partial^2 B_{BE} / \partial r^2 + 2\partial P_{BE} \partial B_{BE} / \partial r^2)\Delta r^2$$

However,

For dM_F, the $\partial P_{BE} / \partial r, \partial^2 P_{BE} / \partial r^2$ are zeros since the price is fixed for all scenarios.

Hence, $\partial M_H / \partial r$ for *H equals* $\partial M_{FE} / \partial r + B_{BE}\ \partial P_{BE} / \partial r + P_{BE} \partial B_{BE} / \partial r$ *and for F,*

$$\partial M_F / \partial r = \partial M_{FE} / \partial r + P_{BE} \partial B_{BE} / \partial r$$

Since $\partial P_{BE} / \partial r < 0$ *and* $\partial M_H / \partial r < 0,$ we have $\partial M_H / \partial r < \partial M_F / \partial r < 0$. Thus, duration of F is shorter than that of H.

By a similar logic, since $\partial^2 P_{BE} / \partial r^2 < 0,$ convexity of F is better than convexity of H.

Appendix D. Calculation of Duration for the Forward Contract and the Back-End Forward

We perform all our valuations in this section at 25 OAS and 100% of the prepay model, as of July 13, 2001.

$H = FE + BE$ [33]

Because we can compute values for H and BE, we can plug in the values for FE in the following equations. (The plugged-in values are underlined.)

$100.926 = \underline{89.3461} + 11.580$ for market value

$100.926*2.221 = \underline{89.3461}*9.756 + 11.580*(-55.921)$ for duration

BE has a negative duration similar to an IO, because the remaining balance drops with decreasing interest rates.

Again,

$F = FE + FWD$

Because we now know the market value and duration of F and FE, we can plug in the values to compute the market value and duration of FWD.

$101.184 = 89.3461 + \underline{11.838}$ for market value

$100.184*1.882 = 89.3461*9756 + \underline{11.828*(-57.549)}$ for duration

Although FWD has a negative duration similar to BE, B = BE - FWD has a positive duration of [11.580(-55.921) - (11.828*(-57.549))]/0.26 = 127. FWD has a longer negative duration compared to that of BE. For FWD, the market value swings more, since the forward price is static and does not rise with falling interest rates (as happens for the market value of BE, where rising prices with falling interest rates cushion the impact of the decreasing remaining balance to some extent).

[33] Yield Book™ has this structure modeled as H: SA1.FE A1, F: SA1. FE A1 with call option, and BE: SA1. FE A2.

ADDITIONAL INFORMATION AVAILABLE UPON REQUEST

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